Ergodicity as a non-binary property

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Abstract

Traditionally, ergodicity is defined as a binary property. This definition, useful in theoretical matters, is useless for decision-making in the real world, as almost all real-world processes are non-ergodic. This paper introduces the concept of ergodicity as a non-binary property and suggests a few real-world applications.

Introduction

Traditionally, ergodicity is defined as a binary property, with its usual definition as follows: an activity is ergodic if the outcome of it being performed n times by a single actor (the *time average*) coincides with the outcome of n actors performing it once (the *population average*).

However, this definition isn't very useful for decision-making in the real world, where almost all processes are non-ergodic (because of the widespread availability of sources of irreversibility). In the real world, I don't care knowing that all my investments are non-ergodic: that's not useful. Instead, I want to know *how much* non-ergodic they are.

Hence the need for a definition of ergodicity as a non-binary property.

Ergodicity as a non-binary property

Let's take two activities, crossing the street and playing Russian Roulette. Both activities are non-ergodic because for $n \to \infty$, the outcome of n people performing the activity once is a few dead and most surviving, whereas the outcome of a single person performing the activity n times is them ending up dead.

However, in the real world, no one performs an activity an infinite number of times. Most of us manage to cross many streets during our lifetime without dying. For most of us, crossing the street looks like an ergodic activity. Conversely, there's no such thing as a professional Russian Roulette player because everyone who attempts it ends up dead, sooner or not much later.

Hence, it makes sense to say that crossing the street is *more ergodic* than playing Russian Roulette.

Similarly, no one keeps an investment for an infinite number of years. Most of us have a finite time horizon, usually a few decades at most. We aren't interested in knowing that, eventually, our investment will go bankrupt. We know that. We want to know how likely it is to go bankrupt *within our time horizon.*

For decision-making in the real world, it's necessary to consider ergodicity a non-binary property.

Suggested definitions

Short definition:
Ergodicity is a property expressing *how similar* the time average and population averages are over relevant time horizons.

Long definition:

Ergodicity is a property of an actor, an activity, and the relationship between the two, and expressing *how similar* the time average and population averages are over time horizons of relevance to the actor.

Quantitative and qualitative methods

It is tempting to use quantitative methods to estimate ergodicity as a non-binary property – for example, computing the ratio between the time and population averages (and perhaps applying a logarithm).

However, these methods can only be applied to real-world considerations where there is a high certainty of knowing all relevant variables and their values. This is seldom the case. Do we really know the hazard rate for this person or all the risks associated with that investment?

Qualitative methods, while less precise in theory, have fewer blind spots and therefore are often more effective in practice.

Conclusions

For decision-making in the real world, ergodicity should be considered a non-binary property. Two questions that matter are how likely the activity is to behave non-ergodically within time horizons of relevance and how much non-ergodically it would behave.

Further readings

Luca-dellanna.com/what-is-ergodicity

Ergodicity: How irreversible outcomes affect long-term performance in work, investing, relationships, sport, and beyond (3rd edition)

Ergodicity Economics (Peters and Adamou, 2017)